Corresponding states of fluid motion in nematic liquid crystals

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We study the dynamic origin of the corresponding states law for the one-dimensional and magnetic walls found above the Fréedericksz threshold. The study shows that (i) the absolute values for the dynamic parameters of the nematic liquid crystals, such as their viscosity coefficients, have secondary importance in the determination of the observed universality and (ii) the elastic constants and some ratio between the viscosity coefficients, which remain practically constant when the nematic material is changed, are the most important factors in determining the walls' periodicity. [S1063-651X(99)02501-5]

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I. INTRODUCTION

One of the most interesting aspects of many-body physics is that compounds that are completely different from the microscopic point of view can reveal, after appropriate scaling, essentially the same macroscopic behavior. For example, close to a critical point, the thermodynamical behavior of very distinct microscopic systems can be put along a single line of corresponding states [1]. The classic example of this behavior is the unique coexistence curve that can be obtained when the temperature and density of different fluids are scaled by their critical values [2,3]. This kind of curve provides powerful insight into the nature of these systems. All the particular parameters of the systems may be put aside and all that remains is a universal behavior with the essential features of some large thermodynamical class.

It has recently been observed that some geometrical parameters of the one-dimensional and periodic nematic liquid crystals (NLC) magnetic walls produced above the Fréedericsz threshold can be put along a common line of corresponding states [4]. The static approach has given an unsatisfactory explanation for this universality even when the parameters of the walls are measured in a static situation. As these walls arose from a complex fluid flow [5], their macroscopic parameters may be a function of the physics happening at the period at which the walls are created and therefore the observed regularity must also be a sign of some kind of universality existing in the course of their dynamic birth. If proved, this would be a very interesting finding and an indication that not only the static parameters belong to a universal class, but also the dynamic ones.

The purpose of this paper is to understand how this universal character occurs. Two distinct features of the physics of the walls are studied: the static (indeed quasistatic [6]) presented when the walls' parameters are measured and the dynamic physics that governs their origin [5,7,8]. The reasons why their static properties suggest a universality was discussed in a previous work [4]. In this paper the dynamic origin of the corresponding states is emphasized. Our most representative result is that the absolute values of the viscosity coefficients have secondary importance in the determination of the walls' parameters. Highly significant, as it will be shown, is the existence of a nearly constant ratio between the viscosity coefficients [9,10].

II. STATIC CORRESPONDING STATES

For a broad class of oriented NLCs with positive diamagnetic anisotropy, the action of an external magnetic field may lead to the formation of one-dimensional and periodic structures, named walls [11,12]. These structures make the transition between adjacent symmetrical distorted regions [12] and have been widely investigated from both theoretical and experimental points of view [5–9,13–16]. Their practical importance, beyond being typical examples of textures in NLCs, lies in the fact that with simple experiments the measurement of their parameters can provide elastic constants values as well as magnetic susceptibility [17].

In order to obtain these structures, a NLC sample inside a microslide glass with dimensions (a,b,d) that satisfy the relation $a \gg b \gg d$ is used. The director is initially uniformly aligned along the $\vec{e}_x$ direction and an external controlled magnetic field H is applied along the $\vec{e}_y$ direction. While this is done, there is a competition between the magnetic susceptibility, which tends to align the director along the magnetic field direction, and the elastic energy, which tends to produce a director orientation consistent with its orientation at the surface of the sample. When H is greater than the Fréedericsz threshold H_c, the magnetic susceptibility overcomes the elastic resistance of the medium and the director begins to bend towards the external magnetic field in order to become oriented either parallel or antiparallel to it. Due to this symmetry breaking, the bending of the director is not homogeneous in the sample [11,12]. The presence of a set of unidimensional structures, walls, periodically distributed along the $\vec{e}_x$ direction is observed.

From the experimental data it is possible to connect the wavelength of the periodic distortion λ to the applied magnetic field [17,18]. That is, for each different magnetic field H applied to the sample, walls with different wavelengths λ appear. Therefore, when a plot of the wavelength λ is created as a function of the applied magnetic field H, a different curve for each different NLC sample is obtained. Nevertheless, when using a change of scale that depends only on the elastic constants of the material, we have discovered, in a previous work [4], that all these distinct curves can be joined together.

As the external magnetic field is applied along the direction perpendicular to the initial orientation of the director, we

can suppose that the director will always remain in the plane defined by the magnetic field direction and the initial orientation of the director [5], that is,

$$n_x = \cos\,\theta(x,y,z), \quad n_y = \sin\,\theta(x,y,z), \quad n_z = 0, \quad (1)$$

where $\theta(x,y,z)$ is the angle between the director $\vec{n}$ and the $\vec{e}_x$ direction. There is a scaling in the geometry defined in Eq. (1) that transforms the elastic free energy to an expression without any parameter representing a unique NLC sample. This can be understood by considering the two elastic constant approximation of the Frank free energy in the presence of an external magnetic field [12,13]

$$F = \int_V \left\{ \frac{1}{2} K_{33}[(\partial_x \theta)^2 + (\partial_y \theta)^2] \right.$$
$$\left. + \frac{1}{2} K_{22}(\partial_z \theta)^2 - \frac{1}{2} \chi_a H^2 n_y^2 \right\} dV, \quad (2)$$

where K_{11}, K_{22}, and K_{33} are the elastic constants of splay, twist, and bend, respectively, and V is the volume of the sample.

For the system described by Eq. (2), strong anchoring boundary conditions are assumed and, as the walls are one-dimensional structures extending along the $\vec{e}_x$ direction, the simplest director's configuration along the $\vec{e}_y$ and $\vec{e}_z$ directions [19] will be considered. That is,

$$\theta(x,y,z) = \eta(x) \sin\left(\frac{\pi y}{b}\right) \sin\left(\frac{\pi z}{d}\right), \quad (3)$$

where the field $\eta(x)$ describes the configuration of the director along the $\vec{e}_x$ direction.

With these hypotheses and the definitions of the variables h and $\bar{x}$ by the relations

$$\chi_a H_c^2 = K_{33}\left(\frac{\pi}{b}\right)^2 + K_{22}\left(\frac{\pi}{d}\right)^2,$$

$$x^2 = \frac{K_{33}}{\chi_a H_c^2}\,\bar{x}^2,$$

$$h = \frac{H}{H_c}, \quad (4)$$

the free energy (2) becomes

$$F = \frac{1}{4} bd \sqrt{K_{33}(\chi_a H_c^2)} \int_0^{\bar{L}} \mathcal{F}\,d\bar{x}, \quad (5)$$

where

$$\mathcal{F} = \frac{1}{2}(\partial_{\bar{x}} \eta)^2 + \frac{1}{2}\eta^2 - 2h^2 u(\eta),$$

$$\quad (6)$$

$$u(\eta(\bar{x})) \equiv \int_0^1 \int_0^1 d\bar{y}\;d\bar{z}\;\sin^2[\,\eta(\bar{x})\sin(\pi\bar{y})\sin(\pi\bar{z})].$$

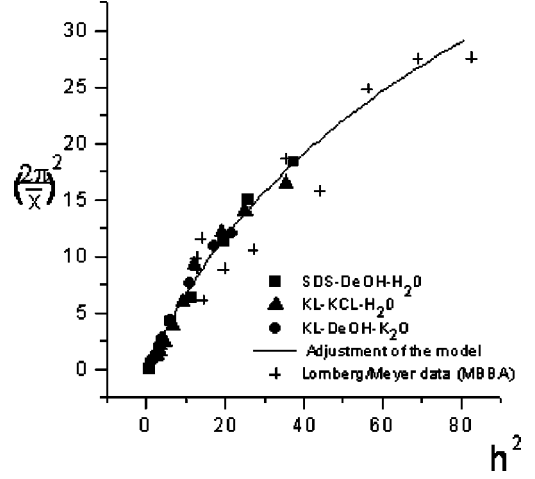$\bar{L}$ is the rescaled sample length along the $\vec{e}_x$ direction, $\bar{y} = y/b$, and $\bar{z} = z/d$.

FIG. 1. Behavior of $(2\pi/\bar{x})^2$ as a function of h^2 for three lyotropic substances and one thermotropic substance (MBBA) obtained from [5]. For the MBBA the data reported in [5] were used. h^2 and $\bar{x}$ are dimensionless quantities.

As stressed in a previous work [4], all the material parameters characterizing any particular NLC sample have been put aside. Therefore, one expects that all static configuration of the NLC can be reduced to a line of corresponding states [1], described by the universal (free from material parameters) equation

$$\partial_{\bar{x}}^2 \eta - \eta + 2h^2 u'(\eta) = 0. \quad (7)$$

When the scaling laws (4), were applied to experimental data of different samples it was verified that all measured points were extended along the same universal line [4]. In Fig. 1 the result of such scaling applied to the data of some different nematic sample is shown.

Despite the static arguments presented above, the results shown in Fig. 1 are unexpected. The elastic properties of the NLC material are not solely responsible for the geometry of the walls. In a remarkable work, Lonberg et al. [5] showed how both the combined action of the external field on the director and the matter movement produce the walls regularities: The external magnetic field rotates the director, which stimulates a fluid flow generating a nonuniform rotation pattern of the director and reinforces opposite rotations in the samples' neighboring regions. Therefore, it may expected that some dynamic parameters, such as the viscosity coefficients, will contribute to the determination of the geometrical properties of the walls.

It is with some surprise that one discovers that the static parameters, through the scaling presented above, are enough to put the experimental data along a single curve. Consequently, the success of the scaling laws in reducing these experimental data to a unique universal line seems to indicate that there is also some unknown universality in the dynamic period at which the walls are built. In the next section we discuss this problem.

III. DYNAMIC CORRESPONDING STATES

In order to study the NLC dynamics, the so-called Eriksen-Leslie-Parodi (ELP) approach [20–24] is used. Thus

the time evolution of the director direction and the motion of the nematic material will be given by a set of differential equations composed of the anisotropic version of the Navier-Stokes equation [25]

$$\rho\left(\frac{\partial V_\alpha}{\partial t}+v_\beta\frac{\partial V_\alpha}{\partial x_\beta}\right)=\frac{\partial}{\partial x_\beta}(-p\,\delta_{\alpha\beta}+\sigma_{\beta\alpha}), \qquad (8)$$

the balance of torques equation, which in the geometry fixed above assumes the form [19]

$$\gamma_1\partial_t\theta=\gamma_1 W_{xy}-\gamma_2[A_{xy}(n_x^2-n_y^2)+(A_{yy}-A_{xx})n_x n_y]$$
$$+K_{33}[\partial_x^2\theta+\partial_y^2\theta]+K_{22}\partial_z^2\theta+\chi_a H^2 n_x n_y, \qquad (9)$$

and the equation of continuity [26,27]

$$\partial_\alpha V_\alpha=0. \qquad (10)$$

In these equations, ρ is the density of the system, V_α is the α component of velocity, p is the pressure, and $\sigma_{\beta\alpha}$ is the associated anisotropic stress tensor [27], which is dependent on variables such as the velocity $\vec{V}$ of the fluid, the bending of the director θ, and the director time variation rate $\dot\theta$. In Eq. (9) the inertial terms were not considered, γ_1 and γ_2 are the shear torque coefficients, $A_{\alpha\beta}=\frac{1}{2}(\partial_\alpha V_\beta+\partial_\beta V_\alpha)$, and $W_{\alpha\beta}=\frac{1}{2}(\partial_\alpha V_\beta-\partial_\beta V_\alpha)$.

This set of differential equations constitutes a strong coupled nonlinear system and to solve it some approximations must be made. For the first approximation it is assumed that the fluid velocity does not have components along the $\vec{e}_z$ direction [28,29]. Only the $\vec{e}_x$ and $\vec{e}_y$ components of Eq. (8) will be considered, that is,

$$\rho\frac{dV_x}{dt}=-\partial_x p+\partial_x\sigma_{xx}+\partial_y\sigma_{yx}+\partial_z\sigma_{zx},$$
$$\qquad (11)$$
$$\rho\frac{dV_y}{dt}=-\partial_y p+\partial_x\sigma_{xy}+\partial_y\sigma_{yy}+\partial_z\sigma_{zy}.$$

Differentiating the first of these equations with respect to y and the second with respect to x and subtracting the second from the first, the pressure p disappears and

$$\rho\frac{d}{dt}(\partial_x V_y-\partial_y V_x)=\partial_x^2\sigma_{xy}-\partial_y^2\sigma_{yx}+\partial_x\partial_y(\sigma_{yy}-\sigma_{xx})$$
$$+\partial_z(\partial_x\sigma_{zy}-\partial_y\sigma_{zx}) \qquad (12)$$

is obtained. This equation governs the flux of the nematic material.

The above approximation is not enough to give us a solution to this equation. The usual way to proceed is to restrict the analysis to the linear terms of the viscosity tensor. As it will be shown later, this procedure overshadows the wall's universal character. Consequently, the above differential equation will not be solved over the entire sample, but just over a very restricted portion of it. This procedure will not eliminate the universal character of these structures, but it will provide a tractable nonlinear version of the differential equation.

It is important to remember that the component V_x of the velocity is relevant only around the sample's edges [5,19]. Thus we restrict the present analysis to the portions of the sample where the V_y velocity component is dominant. Therefore, we make

$$V_x\equiv 0. \qquad (13)$$

This condition being met together with the continuity equation gives $\partial_y V_y=0$.

It will also be considered that the director oscillatory character is present since the beginning of the walls creation. This allows us to assume that, even at these initial moments, there is a set of lines where

$$\partial_x\theta=0. \qquad (14)$$

Finally, the one-dimensional character of these structures requires that, once x is fixed, the director will be constant along the direction of the external magnetic field, that is [19],

$$\partial_y\theta=0. \qquad (15)$$

The three conditions stated in Eqs. (13)–(15) will limit this study to the sample regions where the director shows its greatest amplitude. In this way Eq. (12) becomes

$$\rho\frac{d}{dt}(\partial_x V_y)=A_1\partial_x^2\theta\partial_x v_y+A_3\partial_x^3 v_y$$
$$+A_4\dot\theta\partial_x^2\theta+A_5\partial_x^2\dot\theta+A_2\partial_z^2\partial_x v_y, \qquad (16)$$

where

$$A_1\equiv A_1(\eta_i,\theta)=2[-\eta_1+\eta_2+\eta_{12}(n_x^2-n_y^2)]n_x n_y,$$
$$A_2\equiv A_2(\eta_i,\theta)=\eta_3+(\eta_2-\eta_3)n_y^2,$$
$$A_3\equiv A_3(\eta_i,\theta)=\eta_1 n_x^2+\eta_2 n_y^2+\eta_{12}n_x^2 n_y^2, \qquad (17)$$
$$A_4\equiv A_4(\eta_i,\theta)=2(\eta_1-\eta_2)n_x n_y,$$
$$A_5\equiv A_5(\eta_i,\gamma_1,\theta)=\frac{1}{2}(\eta_1+\eta_2-\gamma_1)-(\eta_1 n_x^2+\eta_2 n_y^2).$$

In this equation η_1, η_2, η_3, η_{12}, and γ_1 are the five independent viscosities coefficients [27]. It should be observed that as A_i, $i=\{1,\ldots,5\}$, is θ dependent, Eq. (16) is yet strongly nonlinear.

Using these approximations and considering that $\gamma_1\simeq-\gamma_2$, Eq. (9) becomes

$$\gamma_1\partial_t\theta=\gamma_1 n_x^2(\partial_x v_y)+K_{33}[\partial_x^2\theta+\partial_y^2\theta]+K_{22}\partial_z^2\theta+\chi_a H^2 n_x n_y.$$
$$(18)$$

From Eqs. (16) and (18) the universal nature of the walls formed during the fluid motion will be extracted.

Now, in order to further simplify Eqs. (16) and (18), the director periodicity, the strong anchoring hypothesis stated in Eq. (3), and the change of scales already used in the static case in Eq. (4) will be used. At first the wall's periodicity along the $\vec{e}_x$ direction implies that

TABLE I. Viscosities in 10^{-3} kg m^{-1} s, the ratio K_{33}/K_{22}, and the references from which these data were obtained as a function of the NLC compound. The question mark means that the corresponding value is unknown to the authors. The values of the K_{33}/K_{22} accompanied by a question mark are attributed considering the average of the others values. The compound labeled ''mixture'' was found in Ref. [30] and is a 1:1:1 molar mixture of HBAB with p-n-butoxybenzylidene-p-aminobenzonitrile and p-n-octanoyloxybeziledene-p-aminobenzonitrile.

Compound	Temperature (°C)	η_1	η_2	η_3	η_{12}	γ_1	K_{33}/K_{22}	References
MBBA	25	103	23.2	39.6	6	74.6	2.5	[30,31]
CBOOA	100	21.2	5.81	8	10.9	15.41	3.7	[32,33]
HBAB	80	43.3	9.4	14.6	6	32.5	3(?)	[30]
PAA	122	9.3	2.3	3.4	4	6.6	3.1	[26,31]
Mixture	80	65.1	13.4	22	5	48.5	3(?)	[30]
5CB	3(?)	86	17.6	33	?	63.6	2.3	[34,35]

$$\partial_x^2 \theta = -k^2 \theta,$$

$$\partial_x^2 V_y = -k^2 V_y \tag{19}$$

and the strong anchoring condition leads to

$$\partial_y^2 \theta = -\left(\frac{\pi}{b}\right)^2 \theta,$$

$$\partial_z^2 \theta = -\left(\frac{\pi}{d}\right)^2 \theta. \tag{20}$$

Furthermore, as $k = 2\pi/\lambda$, the change of scale in x implies that there is also a change of scale in k given by

$$k^2 = \frac{\chi_a H_c^2}{K_{33}} \bar{k}^2. \tag{21}$$

Finally, a time scaling given by

$$t = \frac{\gamma_1}{\chi_a H_c^2} \tau \tag{22}$$

is introduced. With these replacements and observing that

$$\frac{K_{33}}{K_{22}} \frac{d}{\pi} \frac{\chi_a H_c^2}{\gamma_1} \approx K_{33} \frac{\pi}{d} \frac{1}{\gamma_1} \ll 1, \tag{23}$$

Eqs. (16) and (18) become

$$\partial_\tau \theta = n_x^2 \partial_x \tilde{V}_y - (\bar{k}^2 + 1)\theta + h^2 n_x n_y, \tag{24}$$

$$\partial_x \tilde{V}_y = -R\dot{\theta}, \tag{25}$$

where

$$R \equiv R(\bar{k}^2, \theta) = \frac{\bar{k}^2 (A_4 \theta + A_5)}{(A_1 \theta + A_3)\bar{k}^2 + \dfrac{K_{33}}{K_{22}} A_2}. \tag{26}$$

Consequently, the two dynamical equations (24) and (25) can be put together in a single one as

$$\partial_\tau \theta = \frac{-(\bar{k}^2 + 1)\theta + h^2 n_x n_y}{1 + n_x^2 R(\bar{k}^2, \theta)}. \tag{27}$$

This equation gives the director's time growth rate only at the regions where it achieves its greatest bending. In spite of this limitation, this equation has some advantages. First, it maintains the nonlinear structure of the phenomenon and can be easily integrated numerically. Furthermore, it shows, on the left-hand side, the variation rate of the $\partial_\tau \theta$ term and, on the right-hand side, its dependence on $\tilde{k}^2$, θ, and the viscosity coefficients. According to Lonberg et al., the preferred $\bar{k}^2$ is determined by the value of $\bar{k}^2$ that affords the maximum to $\partial_\tau \theta$. In this equation this can be easily done and, as it will be shown, the result is not a constant and it depends on the θ amplitude. This is not a new finding. Srajer et al. [7], when analyzing the momentum space, showed that the $\bar{k}^2$ exhibited by the system is not the one that gives the maximum to $\partial_\tau \theta$ at the initial moments of the matter flow when $\theta \approx 0$. It is fixed during the motion of the nematic material [8]. This is the reason why we have retained the nonlinear terms in Eq.
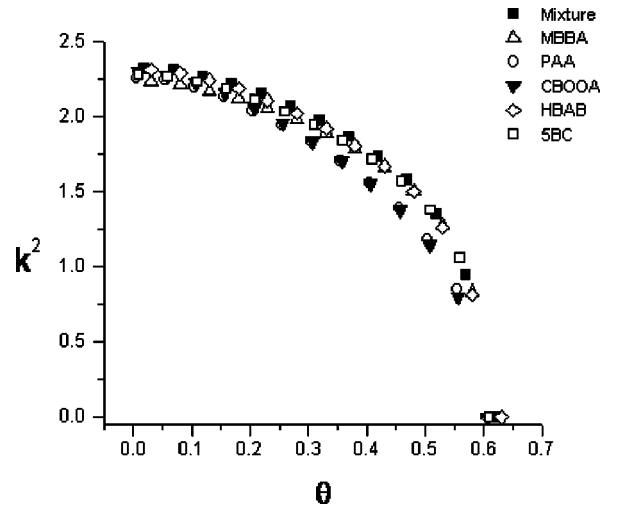

FIG. 2. Reduced $\bar{k}^2$ maximizing the growth rate of the director as a function of the angle θ of the bending of the director [Eq. (27)] for the compounds shown in Tables I and II. The reduced $\bar{k}^2$ was defined in Eq. (21). The reduced magnetic field was fixed at $h^2 = 5$. θ is measured in radians and $\bar{k}^2$ is dimensionless.

TABLE II. Reduced values of the viscosities $\bar{\eta}_i = \eta_i/\eta_3$ $(i=1,2,3)$, $\bar{\eta}_{12} = \eta_{12}/\eta_3$, and $\bar{\gamma}_1 = \gamma_1/\eta_3$, as a function of the NLC compound. The question mark means that the value is unknown and, as discussed in the text, it has not influenced our final result.

Compound	$\bar{\eta}_1$	$\bar{\eta}_2$	$\bar{\eta}_{12}$	$\bar{\gamma}_1$	$\dfrac{\bar{\eta}_1 + \bar{\eta}_2 - \bar{\gamma}_1}{\bar{\eta}_2 - \bar{\eta}_1 - \bar{\gamma}_1}$	$\dfrac{1}{\bar{\eta}_2 - \bar{\eta}_1 - \bar{\gamma}_1}$
MBBA	2.60	0.58	0.15	1.88	-0.33	-0.51
CBOOA	2.65	0.72	1.36	1.92	-0.38	-0.52
HBAB	2.96	0.64	0.41	2.23	-0.30	-0.44
PAA	2.73	0.67	1.18	1.94	-0.36	-0.50
Mixture	2.95	0.61	0.23	2.20	-0.30	-0.44
5CB	2.60	0.53	?	1.92	-0.30	-0.50

(27). However, the aim of this work is not to understand how the λ presented by the system is selected among the possibilities provided by the criterium of Lonberg et al., but to show how this choice assumes a universal character. Consequently, we have evaluated the curve of the preferred $\bar{k}^2$ as a function of θ (for each θ) and showed that the result presents elements of universality. By universal behavior it is understood that as the nematic specimen is changed, the same behavior, after the appropriated mapping, is observed in all compounds.

In order to realize this task, we look for some NLC characterizing parameters and use them in Eq. (27). These NLC compounds, their viscosities coefficients, and the references from where they were obtained are displayed in Table I.

To each of these compounds, values of $\bar{k}^2$, as a function of θ, that make Eq. (27) a maximum have been found numerically. These are the preferred $\bar{k}^2$. An outstanding aspect of this finding, shown in Fig. 2, is that for samples as distinct as the ones shown in Table I, the curves of the preferred $\bar{k}^2$, as a function of θ, are not so different. Only the elastic constants have been scaled and the viscosities, which are explicitly present in Eq. (27), seem not to play an important role. This finding justifies the previous result found in the corresponding states of periodic structures of NLCs [19]. However, how could this happen? What mechanism makes the viscosity coefficients become practically absent in the preferred $\bar{k}^2$ curve?

First, Eq. (17) is rewritten as

$$A_1 = -A_4 + 2n_x n_y \eta_{12}(n_x^2 - n_y^2),$$

$$A_3 = -A_5 + \frac{1}{2}(\eta_1 + \eta_2 - \gamma_1) + \eta_{12}n_x^2 n_y^2. \tag{28}$$

Therefore,

$$(A_1\theta + A_3)\tilde{k}^2 + \frac{K_{33}}{K_{22}}A_2 = -(A_4\theta + A_5)\tilde{k}^2$$

$$+ \eta_{12}n_x n_y[2(n_x^2 - n_y^2)\theta + n_x n_y]\tilde{k}^2$$

$$+ \frac{1}{2}(\eta_1 + \eta_2 - \gamma_1)\tilde{k}^2$$

$$+ \frac{K_{33}}{K_{22}}[\eta_3 + (\eta_2 - \eta_3)n_y^2] \tag{29}$$

is obtained, which affords

$$R(\tilde{k}^2, \xi_i, \theta) = \frac{\tilde{k}^2(A_4\theta + A_5)}{(A_1\theta + A_3)\tilde{k}^2 + \dfrac{K_{33}}{K_{22}}A_2} = \frac{\tilde{k}^2}{-\tilde{k}^2 + S(\tilde{k}^2, \eta_i, \theta)}, \tag{30}$$

where

$$S(\tilde{k}^2, \eta_i, \theta) = \frac{\eta_{12}n_x n_y[2(n_x^2 - n_y^2)\theta + n_x n_y]\tilde{k}^2 + \frac{1}{2}(\eta_1 + \eta_2 - \gamma_1)\tilde{k}^2 + \frac{K_{33}}{K_{22}}[\eta_3 + (\eta_2 - \eta_3)n_y^2]}{[2(\eta_1 - \eta_2)n_x n_y]\theta + \frac{1}{2}(\eta_1 + \eta_2 - \gamma_1) - (\eta_1 n_x^2 + \eta_2 n_y^2)}$$

$$= \frac{\bar{\eta}_{12}n_x n_y[2(n_x^2 - n_y^2)\theta + n_x n_y]\tilde{k}^2 + \frac{1}{2}(\bar{\eta}_1 + \bar{\eta}_2 - \bar{\gamma}_1)\tilde{k}^2 + \frac{K_{33}}{K_{22}}[1 + (\bar{\eta}_2 - \bar{\eta}_3)n_y^2]}{[2(\bar{\eta}_1 - \bar{\eta}_2)n_x n_y]\theta + \frac{1}{2}(\bar{\eta}_1 + \bar{\eta}_2 - \bar{\gamma}_1) - (\bar{\eta}_1 n_x^2 + \bar{\eta}_2 n_y^2)}, \tag{31}$$

where $\bar{\eta}_1 = \eta_1/\eta_3$, $\bar{\eta}_2 = \eta_2/\eta_3$, $\bar{\eta}_{12} = \eta_{12}/\eta_3$, and $\bar{\gamma}_1 = \gamma_1/\eta_3$ are the reduced viscosities. Table II shows the values of these reduced parameters.

The reason for the coincidence of the preferred $\bar{k}^2$ curves become clear in Eqs. (30) and (31) and Table II. It is not the absolute values of these viscosity coefficients that are important, but the relative ones. When compared to the dispersion of the absolute values, the relative ones are much more coincident. For example, in Table I the values for η_1 range from 9.3 to 103×10^{-3} kg m^{-1} s. Meanwhile, in Table II they range from 2.60 to 2.96. It is this small variation of the viscosity coefficient relative values that is responsible for the corresponding states of the NLC walls.

It is important to emphasize that although the relative values of the viscosity coefficients are much more coincident than the absolute ones, they are not exactly the same (see Table II). Therefore, one can expect to find the presence of the viscosity coefficients in the macroscopic parameters of the wall through a little effect. However, the present data for the wall's periodicity λ are not sufficient to reveal it and we can affirm that, if observed, such an effect will be very small. It should be noticed that even being undetermined at the moment at which the matter flow begins, the value of λ must be fixed when the bending of the director is small. When this condition is met in Eq. (31) the parameters determining the preferred $\bar{k}^2$ curve are given by the ones presented in the last two columns in Table II. In this case the values' coincidence is indeed much more surprising.

Therefore, we see that the key to understanding the law of the corresponding states in the nematic state is the existence of an approximate constant ratio between the viscosity coefficients. A report of such a constant relation between the viscosity coefficients is found in [10]. They found, examining their experimental data, some *ad hoc* laws connecting the values of the Miesowicz viscosity coefficients. They do not give any justification for the existence of this constant ratio. We believe that, as in the seminal work of Helfrish [9], it is related to the anisotropic shape of the nematic micelle.

IV. CONCLUSION

This study searched for the origin of the law of corresponding states for the NLC walls produced above the Fréedericksz threshold [4] and, since the static approach is not enough to give an explanation for this experimental law, a dynamic explanation is presented. A change of scale, similar to the one used in the static case, was applied to the ELP set of differential equations and the viscosities coefficients were not scaled out. Despite the large variations of the viscosity coefficients, when the nematic specimen is changed, the curve giving the preferred $\bar{k}^2$ as a function of θ undergoes little changes. As regards the reason for this behavior, it was found that the relative values of the viscosity coefficients are the really important factors in determining the walls' periodicity. Furthermore, these relative values are much more coincident than their absolute values. We conjecture that these near ratio coincidences are related to the factorization of theses coefficients in a geometrical factor and in a function representing the internal degrees of freedom in the NLC micelles [9]. When the ratio between the viscosity coefficients is performed, the internal degrees of freedom are taken off and all that remains is the ration between the geometrical factors. Therefore, when the nematic material is changed, this number is much more coincident than the viscosity coefficients as a whole.

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